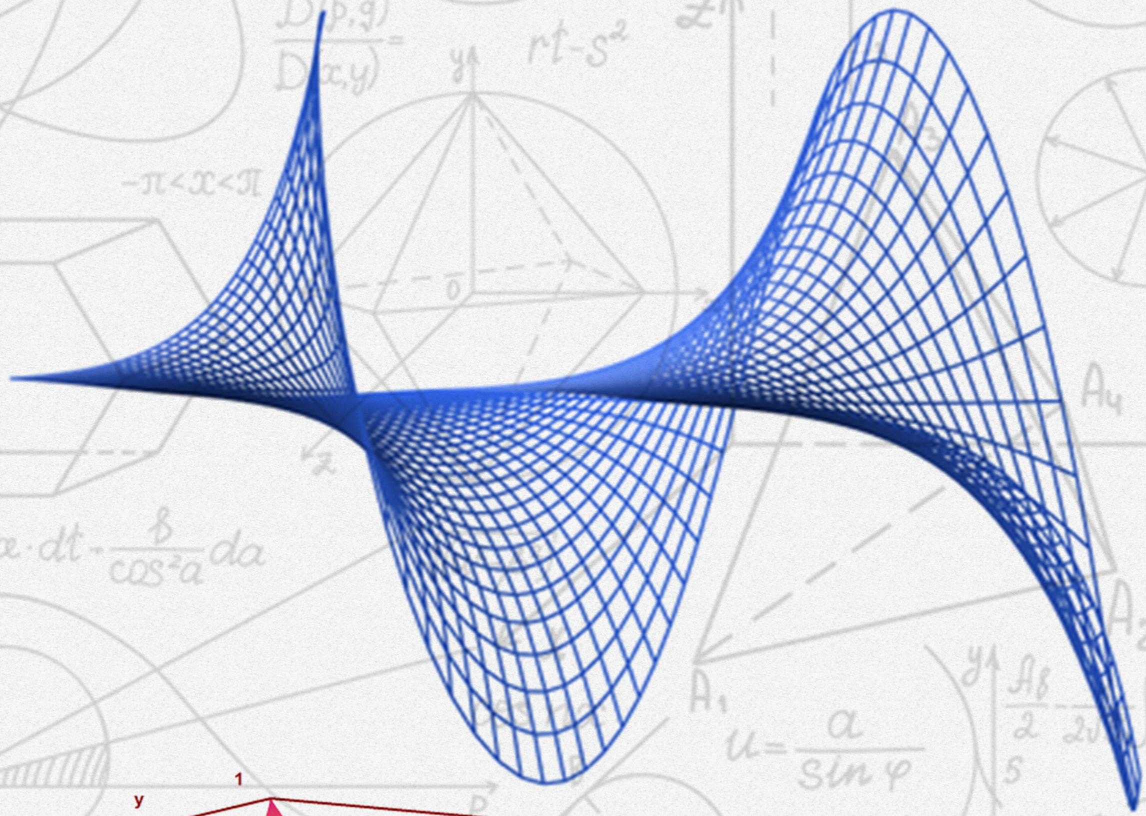
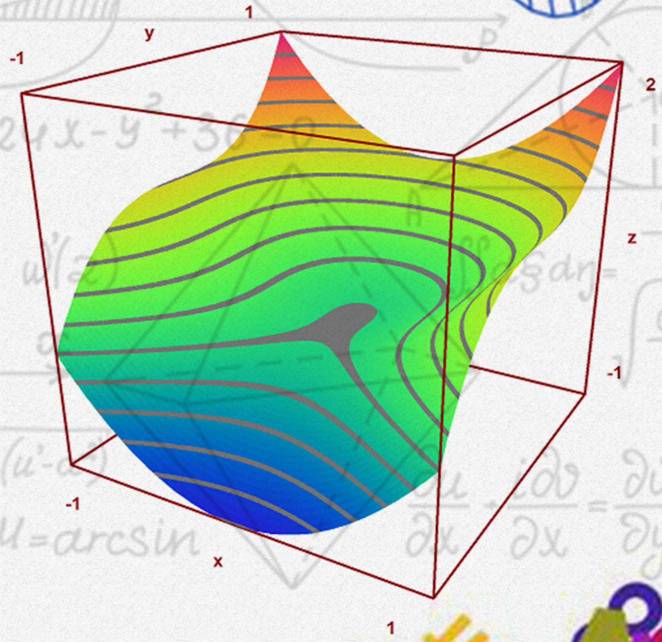
Applied Mathematics And Humanities Department
S.V.N.I.T.

# A MaThing

Second Edition






$$e^{i\pi} + 1 = 0$$

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# Message from HoD

*Dr. Sushil Kumar Prajapati*

*It is heartening to note that, the Applied Mathematics & Humanities Department is ready with the 2nd issue of our E-Magazine, 'AMaThing'. The variety and creativity of the articles in the magazine represents that our faculties and students are not only pursuing academic excellence, but they are also good thinkers, creative and innovative writers. I congratulate all contributors and the editorial group for their hard work and dedication in bringing out the magazine. I appreciate the devotion and commitment of the entire team. 'All the Best'*

-x-x-x-

# Lotfi Aliasker Zadeh: Father of Fuzzy Set Theory

*Dr. Dhananjay Gopal,*
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Son of an Iranian press correspondent and a Russian mother, a doctor by profession, Lotfi A. Zadeh was born on 4th February, 1921 in Baku, Azerbaijan (former USSR). He completed his elementary schooling in Azerbaijan. Later he shifted to Tehran in 1931. At the age of thirteen, he constructed a rotary motor that was simpler than the Wankle motor engine. In Tehran, he enrolled at Alborz college, an American missionary school where he met Fay, who later became his wife. He graduated in 1942 from Tehran University in Electrical Engineering, joined MIT in 1944 and in 1946, obtained his Masters degree. In 1949, he was awarded a doctorate from Columbia University, USA, for his work on "circuits and systems on frequency domain analysis of time-variable network".

In 1950, he published two articles, "*Thinking Machines: A new field of Electrical Engineering*" and "*An Extension of Wiener's Theory of Prediction*", which could be considered, respectively, as forerunners of *Artificial Intelligence* and *Systems Theory*; he coined the latter in another article published in the *quarterly Columbia magazine* in 1954. During that period, he spent some time at the Institute of Advanced Study, Princeton. Dr. Zadeh moved to University of California, Berkeley in 1959 and began work on optimal control, time-varying systems, systems identifications and other areas. During his period as the chairman of the Electrical Engineering Department at Berkeley, he merged it with the Computer Science Department. He formed a new department that was named as Electrical Engineering and Computer Science Department. It is now one of the world's top ranked Computer Science Departments. This idea of a joint EECS department was adopted in the upcoming days by many other well-known universities and institutions.

Around 1963, while studying *Linear Systems Theory*, he realized that the formalization of linear systems could not go beyond a certain level of rigorous precision, which once surpassed rendered conclusions of little relevance. In fact, he wanted to classify objects or sets having curved boundaries which could be used for approximations of some real measurements. Despite difficulties, he found a solution for this problem and published his famous 1965 paper *Fuzzy sets* in *Information and Control* journal. This article today is one of the citation classics and is perhaps an epoch-making article. Google Scholar presents close to twenty lakh results for the query "Fuzzy Sets", and his work had been cited a remarkable 2,37,373 times as of March, 2020.

It is interesting to note that Garrett Birkhoff, another famous mathematician of the time (best known for lattice theory and quantum logic) had first criticized and expressed disbelief over Zadeh's theory, but later praised it. The main reason for his skepticism was that the new theory challenged other basic mathematical formulations which were the backbone of artificial intelligence, like probability. This criticism was embraced by Zadeh with an open heart and fearlessness, and he further proved that this proposed theory was much better equipped and more able to deal with real-world problems and approximate reasoning. By now, fuzzy logic and fuzzy mathematics have had countless successful applications. To name a few, we have the automated "intelligent" train system in Sendai, Japan, washing machines, vacuum cleaners and expert systems for diagnosis.

In the last decade of the 20th century, Zadeh introduced a new paradigm in the field of computation with his concept of "Soft Computing" as a hybridization of methodologies from fuzzy logic, neural networks and probabilistic reasoning. Today, based on Zadeh's novel ideas, a lot of work is ongoing in these fields to develop applications to make human life more comfortable and benefit the public and the society.

Dr. Zadeh received many honours and awards, which include fellowships of NAE (Nation academy of Engineering), IEEE

(Institute of Electrical and Electronics Engineers), IFSA (International Fuzzy Systems Association), ACM (Association for Computing Machinery), about 24 honorary doctorates from different universities in several countries and many other awards. He published more than 200 papers as a single author and he was active, remarkably, in his work and innovation even at the ripe age of 96.
On September 6th, 2017, Dr Zadeh breathed his last at his home in Berkeley, California. He was 96. He was buried in the first Alley of Honour in Baku, his birthplace. Highly respected people like the President of Azerbaijan attended his funeral.

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# Millennium Problems : Most Difficult Way to Earn a Million Dollars

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One of the hardest ways to make a million dollars is to solve one of the Millennium Prize Problems. The Millennium Prize Problems are seven problems in mathematics that were stated by the Clay Mathematics Institute on May 24, 2000. The problems can be divided, very roughly, into two algebraic problems, two topological problems, two in mathematical physics, and one problem in the theory of computation. The problems are:

- P versus NP
- Hodge Conjecture
- Riemann Hypothesis
- Yang–Mills Existence and Mass Gap
- Navier–Stokes Existence and Smoothness
- Birch and Swinnerton-Dyer Conjecture
- Poincaré Conjecture

Out of the seven problems, six remain unsolved till date.

## Poincaré Conjecture

The only Millennium Problem that has been solved is the Poincaré Conjecture, a problem posed in 1904 by the French mathematician Henri Poincaré. This problem was one of the most essential questions in topology.

*Topology,* which is also known as *Rubber-sheet geometry,* is the mathematical study of the properties that are preserved through deformations, twistings, and stretchings of objects. Tearing, however, is not allowed. In the topology of two dimensions, there is no difference between a circle and a square. (So a topologist is a person who cannot tell the difference between a coffee mug and a doughnut.)

A *manifold* of dimension $n$ is a geometric object equipped with a topological structure such that every point has a neighbourhood that looks like (*is homeomorphic to*), $n-$dimensional Euclidean space, for some $n$. The standard example is a sphere, the surface of a ball embedded in three-dimensional space. A person standing in a football stadium thinks that he is standing on flat ground, as the curvature of the earth (i.e., sphere) is not observable locally. So a sphere is a 2-manifold; the flat ground looks like 2-dimensional Euclidean space. Another example of a 2-manifold is a doughnut (a one holed object).

But the above two manifolds are considered as being different, as one cannot be continuously deformed into the other. Another way to see that the doughnut is different from the 2-sphere is that any loop on the sphere can be contracted to a point. Imagine a rubber band on the surface of a ball; it can be pulled to the top of the ball without breaking the band or leaving the ball. On the contrary, loops on a doughnut cannot be contracted; i.e., the rubber band would either leave the doughnut or you would need to cut it.

The Poincaré Conjecture states that any closed (boundaryless) $n-$manifold which is homotopy equivalent to the $n-$sphere must be the $n-$sphere. This can be proved easily for $n = 1, 2$. Also, it was proved for $n \geq 5$ by Stephen Smale in the 1960s, and for $n = 4$ by Michael Freedman in 1982. Both mathematicians were given Fields Medals, the highest honour a mathematician can receive.

The case of $n = 3$ is equivalent to the following statement:

Any simply connected closed 3-manifold is the same as the 3-sphere.

As $n = 3$ was the only case left to be proved, this was the statement of the Poincaré Conjecture when it was posed as a Millennium Problem. The official statement of the problem was given by John Milnor. Here, 'closed' means that the object is compact (i.e., the object can be kept in a container and its lid can be closed) and 'simply connected' means that the manifold has no holes (i.e., a loop on its surface can always be contracted to a point). So in simple words, Poincaré Conjecture states that if an object is finite and contains no hole, then it is a sphere (there is no difference between a circle and a square, in topology).

Earlier, people used to think that Earth is flat, but the Greek Mathematicians were able to figure out that the Earth was, in fact, spherical. They even calculated the diameter with high accuracy. Today we know that the Earth is spherical after NASA sent out its spacecraft and took photographs of the Earth, but the Greeks were able to figure this out using mathematics very early. So, is it possible to deduce the shape of the universe without stepping outside of it? That is, living in a 3-dimensional world, could we understand the shape of the universe? Henri Poincaré thought so, and by using the Poincaré Conjecture, you can do so, but it is not practical. But it does say that with mathematics there is no limit to what we can discover. With mathematics, in principle, we can find out the shape of the universe we live in without stepping outside of it. Also, the Poincaré Conjecture addresses the fundamental question in the theory of manifolds, i.e., the classification problem: is there a way to characterize when two manifolds are the same, without having to explicitly write down the map that identifies them? That is, does a set of properties exist such that any two manifolds that share all these properties must be the same?

Grigori Perelman, a Russian mathematician proved the conjecture in 2003, by using ideas of Richard Hamilton from the early 1980s, who had suggested the use of Ricci Flow to solve the problem. The solution was published in a series of papers (three papers) in 2002 and 2003 on arXiv. After his proof was verified, he was awarded the Fields Medal in 2006 and was offered the Clay Millennium Prize in 2010. He refused to accept them, however, saying that his contributions were no more significant than Hamilton's.

While one of the seven Millennium problems is solved, six of them still remain unsolved.

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# Sets and Functions: The Two Pillars of Mathematics

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Although we start learning "mathematics" by counting, the true beauty of the subject is seen only when one is introduced to the concept of sets. At a very early stage, probably during school, one is exposed to a *set* as a collection of objects. This somewhat vague "definition" is, in fact, widely accepted by the mathematical community the world over. Once we have sets, we can now start playing with them and a whole branch of *set theory* awaits us! It is a fact that in higher mathematics, one cannot work without the notion of sets. However, in this article, we shall not discuss them. We would rather talk about the other "pillar", namely *functions*.

Functions, in themselves, have a rich history. Varied perceptions and consequently varied definitions given by many ancient mathematicians tell us how abstract mathematics developed. It is a question, especially from the students, that why should one even care to define functions as we have, in the modern sense, and even if we do so, why should one study them so rigorously? To answer these questions about functions, let us first go back in time and look at how several mathematicians had tried to define the term *function*, and how we finally settled on its modern meaning.

Many (modern) mathematicians think that the concept of functions is intrinsic to our thinking; that is, whenever we are doing some mathematics, we think in terms of functions. Is this true? There is no evidence, nor a way to find out! The main reason is that our predecessors are no longer with us and we are already biased. Therefore, one must be careful while talking about the history of functions. It may seem that even the ancient mathematicians perceived functions in the modern sense; however, we should not credit them with their invention.

Speaking of ancient mathematicians and their perception of functions, we can find tables of powers of natural numbers in Babylonian texts. These tables indeed define "functions" from the set of natural numbers $\mathbb{N}$ to itself. However, this is no reason to think that Babylonians thought of the entries of the table as values of a certain function (in the modern sense). This distinction between inventing functions and accidentally using them is seen in the works of Ptolemy. Ptolemy computed lengths of chords of a circle, which is essentially the computation of values of trigonometric functions. However, the trigonometric functions had not been defined in those times!

Galileo probably understood the concept of functions clearly. While studying the motion of (celestial) objects, he found that certain quantities depend on some other quantities. If we know the value of one, we may be able to compute the value(s) of the other. However, he could never give a formal definition of a function. Even without the formal definition, he constructed a one-to-one correspondence between the set of integers $\mathbb{Z}$, and the set of natural numbers $\mathbb{N}$. We will come to the discussion of one-to-one correspondence (or what is famously called *bijection*) a bit later.

Around the same time, Descartes attempted to associate algebra with geometry by considering curves as equations of two variables. In this representation, Descartes understood the dependence of one of the two variables on the other. However, even Descartes never used the term *function*, nor tried to define a similar concept. Newton followed Descartes and was among the first of mathematicians to obtain a function from an infinite series. In his "definition", he used the terms *fluent* to denote independent quantities, *relata quantitas* to designate dependent quantities, and *genita* to refer to quantities obtained from others using arithmetic.

Leibniz in 1673 used the term *function* for the first time. However, he did not define it as we know today. According to Leibniz, a function was the dependence of geometrical quantities on the shape of the curve. With the introduction of the term *function*, and its somewhat vague but accepted definition, started the study of functions. Euler, while working with Bernoulli, improved this definition by saying that a function was "an analytic expression composed in any way whatsoever of a variable quantity and numbers or constants". This was the first time that the term function had a mathematical meaning associated with it. However, even this definition had its own inconsistencies. Euler himself understood that the same analytic expression could be written in two ways. This could cause confusion, and therefore the definition of function needed refinement.

In 1755, Euler published a book, where he gave a refinement to the definition of function he had introduced earlier. He stated that "*if some quantities depend so on other quantities that if the latter is changed the former undergoes a change, then the former quantities are called functions of the latter quantities*". This solved the problem of one analytic expression having more than one representations. However, even after the introduction of this definition, the mathematical community thought of functions as analytic expressions.

It was probably Dirichlet who thoroughly understood the meaning of the term *function* and its beauty. Dirichlet said that a function could be simply thought of as a rule which assigns to every element in the set, a unique element in some other set. In 1837, using this definition, he gave his famous *Dirichlet's function*, $f : \mathbb{R} \to \mathbb{R}$, given by

$$f(x) = \begin{cases} 1, & x \in \mathbb{Q}, \\ 0, & x \notin \mathbb{Q}. \end{cases}$$

This was the first time that someone had come up with the concept of a function as not being an expression in the independent quantity, but just a rule of assignment.
The mathematical community understood that using this definition, they could not only solve the dilemma of using analytic expressions, but also produce a lot of examples and counterexamples. Such a definition allowed a lot of pathogenic functions to arise, which are now quoted as standard counterexamples. One such example is $f : \mathbb{R} \to \mathbb{R}$, defined as

$$f(x) = \begin{cases} e^{-\frac{1}{x^2}}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

This function is used as a counterexample to show that even if all the derivatives of a function exist, the Taylor's series may not exist. It can clearly be seen that all the derivatives of $f$ are zero at $x = 0$. However, the Taylor's series converges to the function only at a single point ($x = 0$).
The importance of functions in the study of mathematics is clearly seen here. However, in modern mathematics, we are concerned about some select types of functions to which we have given certain names. Of these, the most important are *bijections*. Although we cannot have bijections without the modern sense of a function, it seems that we had intuitively perceived them long ago. Before the introduction of (natural) numbers, humans used to count using pebbles. If one wanted to count the number of livestock, one would simply get a lot of pebbles and start to pile them up: one pebble for each animal. After the animals in possession were exhausted, we would say that we had as many animals as the piled up pebbles.
Essentially, all we did was to assign each animal a pebble (i.e., define a function between the set of animals and the set of pebbles). Now, taking motivation from this primal instinct, we define *bijections* between two sets. We say that a function (in the modern sense) is a bijection if when we start assigning every element of our set to a unique element of the target set, we exhaust all the elements of both the sets simultaneously. In fact, we can say something more! If we can find a bijection between two sets, say $A$ and $B$, we say that $B$ has as many elements as does $A$. While this looks surprisingly trivial, it is quite useful in defining what is called "*cardinality*" of sets. In particular, with the notion of bijections, we can define finite sets, countable sets and uncountable sets.
Besides aiding these definitions, bijections help us work smartly. Bijections form a special type of relations, called *equivalence relations*, which further give rise to equivalence classes. Once we have equivalence classes, it is enough to study one of the elements of the class, and all other elements will have the same properties. Since bijections are essentially used to define *cardinality*, the property under consideration here is the "number of elements". Therefore, whenever one wants to study finite sets (and prove certain theorems for them), one looks at the set $\{1, 2, \cdots, n\}$, which has precisely $n$ elements. Any other finite set with $n$ elements will also satisfy the properties (theorems and results) of this particular set. Similarly, if one wants to study countable sets, one looks at $\mathbb{N}$, the set of natural numbers. Therefore, bijections, in a sense, reduce a lot of work!
The modern definition of a function has given rise to modern Algebra. With the definition of a function as we know it, the usual operations of addition (consequently, subtraction) and multiplication (consequently, division) on the set of real numbers, $\mathbb{R}$, can be thought of as functions. To see this, all we need to observe is that "addition" takes two real numbers and gives another real number. So does multiplication. Therefore, in terms of functions, we have

$$+ : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$$

$$\cdot : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$$

This representation of operations is the basis of concepts that include vector spaces, groups, rings and fields. All in all, the whole of linear algebra and modern algebra depends on the modern definition of functions.
Functions also help us look at some mathematical "objects" in a smart way (as they helped us see cardinality through bijections). First, one must understand what a mathematical object means. Since in the study of (higher) mathematics, one always talks about sets, we must understand that this set can be equipped with certain other properties. These properties are assumed to induce some "*structure*" on the set, making the set a *mathematical object*. For example, we say that a vector space is a set equipped with two operations that satisfy so and so properties. A metric space is a set equipped with a function, which we call *metric*. A topological space is a set equipped with a collection of (open) sets, and so on.
With all these concepts, again, the question of studying each one of them comes into the picture. However, studying literally every such mathematical object seems like foolishness! Again, functions come to save the day. If we look at two objects having the same structure (i.e., they satisfy the same properties), then we can have functions between them. Now, what we want is to be able to construct a function that preserves the structure. That is, if we are talking about algebraic sets (groups, rings, fields, vector spaces, etc.), we would want the function to preserve the operations. If we are talking about topological spaces (or metric spaces), we would want to preserve the open sets. However, even before that, all the preservation is of no use if the "number of elements" in the two sets is not the same. Therefore, we need bijections that preserve the structure. We have given names to such bijections, such as *isomorphisms*, *homeomorphisms*, *isometries*, etc. These "special" functions again form an equivalence relation making the work of studying mathematical objects easier!

# Stable Marriage Problem

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## Introduction

The stable marriage problem is a special problem that was introduced in 1962 in a seminar paper by David Gale and Lloyd Shapley and has interested researchers from different areas including mathematics, economics, game theory and computer science. The speciality about the problem is that there are no numbers involved, no computations, no calculations, no algebra and no calculus; it is completely unlike the mathematics that most of us (undergraduates) have seen till now.

The objective of the stable marriage problem is to find a match between men and women, considering preference lists in which each person expresses his/her preference from amongst the members of the opposite gender. In mathematical terms, the aim is to find a bijective matching between two (non-empty) equal-sized sets of elements given an ordering of preferences for each element. The output match must be stable, which intuitively means that there should be no man-woman pair both of which have the incentive to elope.

The problem can be stated in a more descriptive manner as follows: let there be two sets, where one of the sets contains $N$ men, and the other set contains $N$ women. Each person must rank the members of the opposite gender in their order of preference. The goal is then to create a set of couples with the property that there does not exist any pair of a man and a woman who prefer each other over their respective partners in the set of couples. A set of couples having this property is called a stable matching.

Then, the question to be raised is: is it always possible to arrange marriages stably or to always find a bijective mapping on the given sets with the given order of preference? The answer to this question is yes, according to Gale and Shapley. These researchers proved that it is always possible to solve this problem when the number of elements in both sets is equal. The current research (for more information, see [1]) that is being carried out in this field looks at the problem in which the cardinality of both the sets is different. The current research also talks about the instability caused in the Gale-Shapley algorithm due to a variable number of elements in the sets.

## Gale-Shapley Algorithm

The idea of the Gale-Shapley algorithm (also known as the deferred acceptance algorithm) is to iterate through all free men while there is a free man available. Every free man goes to all women in his preference list according to the order. For every woman he goes to, he checks if the woman is free. If yes, they become engaged. If the woman is not free, then the woman has the choice to either say no to him or dump her current engagement according to her preference list. So an engagement, once fixed, can be broken if a woman gets a better option. In more general terms, the algorithm can be expressed as a sequence of "proposals" from men to women. Throughout the execution, each person is either engaged or free; a man can be free or engaged. However, once a woman is engaged, she can no longer be free again. That does not mean that her fiance's marital status cannot change. A man who is engaged more than once obtains fiances who are less desirable to him, while each engagement brings the woman a more favourable partner. A free woman must accept the first proposal she receives and become engaged to whoever proposes to her. When an engaged woman is proposed to, she compares the proposer and her current fiance, then rejects the one she finds less desirable. If she becomes engaged with the new proposer, her ex-partner will now be free again. Each man proposes to the women on his preference list, in the order in which they appear. He does this until he becomes engaged. However, if a woman decides to break off the engagement, he becomes free again. When this happens, he goes back to his sequence of proposals and proposes to the next woman. The algorithm terminates when everyone is engaged. Upon termination, the engaged couples form a stable matching. The time complexity of the Gale-Shapley Algorithm is $O(N^2)$. In general, the family of solutions to any instance of the stable marriage problem can be given the structure of a finite distributive lattice and this structure leads to an efficient algorithm for several problems on stable marriages. In a uniformly random instance of the stable marriage problem with $N$ men and $N$ women, the average number of stable matchings is asymptotically $e^{-1}N\ln N$. In a stable marriage instance chosen to maximize the number of different stable matchings, this number is an exponential function of $N$. For proof of this algorithm, see [2].

## Applications

Algorithms for finding solutions to the stable marriage problem have applications in a variety of real-world situations, with perhaps the best known of these being in the assignment of their first hospital appointments to graduating medical students. The JEE seat allotment algorithm (for more information, see [3]) is inspired by the single-run deferred acceptance algorithm attributed to Gale and Shapley. Another critical and large-scale application of the stable marriage is in assigning servers to users in a large distributed Internet service. Billions of users access web pages, videos, and other services on the Internet, requiring each user to be matched to one of (potentially) hundreds of thousands of servers around the world that offer that service. A user prefers servers that are proximal enough to provide a faster response time for the requested service, resulting in a (partial) preferential ordering of the servers for each user. Each server prefers to serve users that it can with a lower cost, resulting in a (partial) preferential ordering of users for each server. Content delivery networks that distribute much of the world's content and services solve this vast and complex stable marriage problem between users and servers every tens of seconds to enable billions of users to be matched up with their respective servers that can provide the requested web pages, videos, or other services.

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# It's a Fibonacci Sequence!

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## Fibonacci-like Sequences

We all know what a Fibonacci sequence (a.k.a. Fibonacci numbers) is. It is one of the best-known sequences in mathematics. In brief, it is the sequence $(F_n)_{n\in\mathbb{N}}$ with $F_1 = 1$, $F_2 = 1$ and

$$F_n = F_{n-1} + F_{n-2},\ \forall n > 2. \tag{1}$$

What we mean by a Fibonacci-like sequence is a sequence $(a_n)_{n\in\mathbb{N}}$ with fixed $a_1, a_2 \in \mathbb{Z}$ that follows the "Fibonacci rule", i.e.,

$$a_n = a_{n-1} + a_{n-2},\ \forall n > 2. \tag{2}$$

When we take $a_1 = 1$ and $a_2 = 1$, we get the Fibonacci sequence. However, for other values, it produces a wide variety of sequences to play with.

## Pisano Sequences

Consider any such Fibonacci-like sequence. What happens when one of its terms becomes zero (which is now possible since we are allowing negative integers)? More precisely, what happens if for some $n_0$, $a_{n_0}$ becomes 0? Let's find out!
The next term in this case would be

$$\begin{aligned} a_{n_0+1} &= a_{n_0} + a_{n_0-1} \\ &= 0 + a_{n_0-1} = a_{n_0-1} \end{aligned}$$

If one were to find out the subsequent terms, they would look something like the following

$$(\cdots, a_{n_0-1}, 0, a_{n_0-1}, a_{n_0-1}, 2a_{n_0-1}, 3a_{n_0-1}, 5a_{n_0-1}, 8a_{n_0-1}, \cdots)$$

Ring any bells? Yes, it is the Fibonacci sequence multiplied by a number. We will call such a sequence a Pisano sequence. More specifically, we will call a Fibonacci-like sequence $(a_n)_{n\in\mathbb{N}}$ a Pisano sequence if $\exists\ n_0, K \in \mathbb{N}$ such that $a_{n_0+k} = KF_k\ \forall k \in \mathbb{N}$ .

Why such a peculiar name? Pisano is another name for Fibonacci and I'm bored of writing Fibonacci again and again. Quick fact: The name Fibonacci, short for filius Bonacci ("son of Bonacci"), was made up by by the Franco-Italian historian Guillaume Libri. Pisano means "traveller from Pisa".
Convince yourself that for a Fibonacci-like sequence to be a Pisano sequence, one of its terms must be 0 unless its first two terms are equal.

Now, let's hunt for those zeros in Fibonacci-like sequences.

## Hunt for the Zeros

Our objective is to find out the conditions on $a_1$ and $a_2$ so that the sequence $(a_n)_{n\in\mathbb{N}}$ defined by eq. (2) contains a 0 and is thus a Pisano sequence.

At first, this seems difficult. So we take the opposite approach: let's find out the conditions which guarantee that 0 will not appear in the sequence. One such condition is if two consecutive terms of the sequence have the same sign.

Thus, for a sequence to be Pisano sequence, we know that the first two terms must have opposite signs. We add the conditions that $a_1 > 0$ and $a_2 < 0$, since the other way around generates the same sequence with signs reversed and hence contains 0 if and only if the former sequence contains 0. Also, if $|a_2| > |a_1|$, then $a_3 < 0$ which along with $a_2 < 0$ makes it two consecutive negative terms ensuring that the sequence won't contain a 0.

With this knowledge in hand, let's hunt for the zeros of some Fibonacci-like sequences using MATLAB. We will take all possible combinations of $a_1, a_2 \in \mathbb{Z}$ with absolute values less than 13 and the added condition that $|a_2| \le |a_1|$.

The term $N(i, j)$ counts the index of the first term which is 0 of the sequence $(a_n)_{n\in\mathbb{N}}$ with $a_1 = i$ and $a_2 = -j$. $N(i, j)$ is set to 0 if the sequence does not contain 0.

The output for $n = 12$ was as follows:

```
N =

     3    0    0    0    0    0    0    0    0    0    0    0
     4    3    0    0    0    0    0    0    0    0    0    0
     0    5    3    0    0    0    0    0    0    0    0    0
     0    4    0    3    0    0    0    0    0    0    0    0
     0    0    6    0    3    0    0    0    0    0    0    0
     0    0    4    5    0    3    0    0    0    0    0    0
     0    0    0    0    0    0    3    0    0    0    0    0
     0    0    0    4    7    0    0    3    0    0    0    0
     0    0    0    0    0    5    0    0    3    0    0    0
     0    0    0    0    4    6    0    0    0    3    0    0
     0    0    0    0    0    0    0    0    0    0    3    0
     0    0    0    0    0    4    0    5    0    0    0    3
```

If you look closely at the above output, a pattern emerges. All the diagonal elements contain 3, which is expected. Take a look at the occurrences of 4 on the output; they all lie on a line. Same goes for 5. For higher values of $n$, this observation becomes all the more apparent.

## Finding the Conditions

We take a look at some of the Pisano sequences we have just found.

Consider the sequences that have their $4^{\text{th}}$ term as 0.

$$\begin{gathered}(2,-1,1,0,1,1,2,\cdots)\\(4,-2,2,0,2,2,4,\cdots)\\(6,-3,3,0,3,3,6,\cdots)\\(8,-4,4,0,4,4,8,\cdots)\end{gathered}\tag{3}$$

Take note of the relationship between the $1^{\text{st}}$ terms of the sequences; they are multiples of 2. Same goes for the $2^{\text{nd}}$ (multiples of -1), $3^{\text{rd}}$ (multiples of 1) and subsequent terms.

Consider the sequences with the $5^{\text{th}}$ terms as 0.

$$\begin{gathered}(3,-2,1,-1,0,-1,-1,-2,\cdots)\\(6,-4,2,-2,0,-2,-2,-4,\cdots)\\(9,-6,3,-3,0,-3,-3,-6,\cdots)\\(12,-8,4,-4,0,-4,-4,-8,\cdots)\end{gathered}\tag{4}$$

We observe the same pattern for sequences with the $6^{\text{th}}$ terms as 0.

$$\begin{gathered}(5,-3,2,-1,1,0,1,1,2,\cdots)\\(10,-6,4,-2,2,0,2,2,4\cdots)\\(15,-9,6,-3,3,0,3,3,6,\cdots)\\(20,-12,8,-4,4,0,4,4,8,\cdots)\end{gathered}\tag{5}$$

Another interesting observation is that the second terms of (4) and the third terms of (5) are same as the first terms of (3) except for their sign. Same goes for the third, second and first terms of (5), (4) and (3) respectively.

This makes sense once you consider that if we take out the first term of any sequence in (5), it must have its $5^{\text{th}}$ term as 0, i.e., the resulting sequence must be somewhere in (4) (of course we will have to adjust the sign).

We're now in a position to finally state the main result.

Consider a Fibonacci-like sequence with its $n^{\text{th}}$ term as its first 0, where $n \geq 3$. We construct a new sequence by taking out the first term and reversing the signs of each term (or equivalently, multiplying each term by $-1$). The resulting sequence will have its $(n-1)^{\text{th}}$ term as its first 0. Proceeding in this manner we will finally reach a sequence with its $3^{\text{rd}}$ term as its first 0. This sequence will necessarily be of the form $(K,-K,0,-K,-K,-2K,\cdots)_{n\in\mathbb{N}}$ for some $K \in \mathbb{Z}$.

Using eq. (2), we can reconstruct the original sequence. With each backward step, the sequence will look as follows:

$$\begin{aligned}&(K,-K,0,-K,-K,-2K,\cdots) && (3^{\text{rd}}\text{ term as }0)\\&(2K,-K,K,0,K,K,2K,\cdots) && (4^{\text{th}}\text{ term as }0)\\&(3K,-2K,K,-K,0,-K,-K,-2K,\cdots) && (5^{\text{th}}\text{ term as }0)\\&(5K,-3K,2K,-K,K,0,K,K,2K,\cdots) && (6^{\text{th}}\text{ term as }0)\\&(8K,-5K,3K,-2K,K,-K,0,-K,-K,-2K,\cdots) && (7^{\text{th}}\text{ term as }0)\end{aligned}$$

Look at the first term of each sequence.
*Ring any bells?* Its a Fibonacci sequence!
The desired sequence is then

$$\left(F_{n-1}K, -F_{n-2}K, F_{n-3}K, \cdots, (-1)^{n-1}F_2, (-1)^n F_1 K, 0, (-1)^n F_1 K, (-1)^{n+1} F_1 K, \cdots\right) \quad (n^{\text{th}} \text{ term as } 0)$$

For Fibonacci-like sequences with either of the first two terms zero, it is easy to see that it still follows the above pattern. For those that don't have zero but have first two terms with same absolute values but opposite sign, it follows the above pattern after the zero.

Thus, for a Fibonacci-like sequence to be a Pisano sequence, its first two terms must be of the form $F_n K, -F_{n-1}K$ for some $n \in \mathbb{N}$ and $K \in \mathbb{Z}$.

# Introduction to Machine Learning

*Aman Bakshi,*
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Unless you've been living under a rock, you might have heard of the term Machine Learning in one form or the other. It has quickly created a place for itself in almost all tech-related domains. It is now being used in various sectors including banking, e-commerce, health-care and automotive industry, to name a few. In this article, I'll briefly introduce Machine Learning and then we'll use a simple ML algorithm to solve a real-world problem.

The formal definition of Machine Learning is given as: "A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P if its performance at tasks in T, as measured by P, improves with experience E."

Let's take an example to understand this definition. You are the manager of a bank and want to review loan applications using ML to predict whether a given customer will repay a loan or not. The past dataset given to you has details of customers like age, occupation, current balance, mortgage, etc., along with the labels "will repay" or "will not repay". Here, the experience E is our dataset, the task T is the task of classifying a new customer as "will repay" or "will not repay" and the performance P is measured by how accurate our model is at classification. Now, the model is said to learn, according to the definition, if the classification accuracy increases as we see more and more examples from our dataset.

But how do these algorithms learn? Contrary to how difficult it looks from the outside, it is just basic maths. Let's solve the above problem using an algorithm called Logistic Regression. First, some terminology: the information that we have about a customer (age, mortgage, etc.) are called features $(X)$ and the value that we are trying to predict ("will repay" or "will not repay") is called label or target $(y)$. We build our initial hypothesis $(h(w))$

$$h(w) = \sigma(w_1x_1 + w_2x_2 + \cdots + w_nx_n), \tag{1}$$

where $x_i$'s represent each individual feature and $\sigma(x)$ is called the sigmoid function given by

$$\sigma(x) = \frac{1}{1+e^{-x}}. \tag{2}$$

The output of the sigmoid function is between 0 & 1, which makes it good for classification purposes. The $w_i$'s here are called weights. These weights define our model and are the parameters whose values we want to learn or estimate. Initially, we assign random values to these weights and using a row of $X$ from our past dataset produce an output $\hat{y}$. Now we compare $\hat{y}$ with our original $y$ using a loss function (generally, mean squared error). If this value is small, it means that the weights we had assigned were good enough so as to predict accurate values of $y$, and no change is required. But if the value of the loss function is large, it means we need to update the weights.

So we have to update the weights in such a way that the value of the loss function is minimized. This we do by using gradient descent and update the weights as follows:

$$w_i := w_i - \frac{\partial(loss\ function)}{\partial w_i}. \tag{3}$$

We follow this procedure iteratively for all the rows until the values of the weights do not change much. We consider these as final weights and our model is ready. In future, when we receive the loan application of any new customer (the features), we can simply use (**??**) to predict whether he will repay the loan or not!

This was a very straightforward example. In real-world problems, there are many other factors, but even in the most complex of systems, the underlying principles remain the same.

Have any doubts or want to learn more about Machine Learning? Feel free to contact me at amanbakshi4@gmail.com.

# Fighting COVID-19 with AI, ML and Data Science

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The world has united in a war against a new pandemic, COVID-19. The lives of every single one of us have been affected, and we're doing all in our power to fight this disease. Every person, institution, and field can be seen contributing to the cause, in a way never seen before.

But the one thing that can tip the scales in our favour, is the right use of AI, Data Science and Machine Learning to fight the coronavirus. These are the most powerful technological tools in our arsenal, and they're proving to be useful in the following ways:

**Prediction of the outbreak**
BlueDot is an AI start-up that has developed intelligent systems that can sift through people's data to determine the chances of disease occurrence. BlueDot was one of the first companies to predict the coronavirus outbreak (global spread) in December 2019, which was confirmed in February 2020.

**Role of Big Data and Data Analysis**
Initially, the disease was spreading essentially because people, unaware that they had contracted the COVID-19, were travelling to different countries. All nations thus had a lot of incoming people to test, travel histories to process, and test results to keep track of; a task that was impossible to accomplish manually. Big Data tools were used for this purpose, and the collected data was then analysed to look for potential victims.

**Tracking COVID-19 using GIS**
GIS or Geographic Information Systems technology is being used extensively to fight the spread of the disease. The GIS technology uses social media sites, and detects geographic zones where people are extensively talking about the disease. John Hopkins University has a dashboard that shows cases of coronavirus all over the world, and the current trends of disease spread. These are heat maps of a kind, that help us track the spread of the disease, which would have been practically impossible a decade ago. We can then draw various insights from this information and take quick actions.

**Virus and Vaccine Research**
The need of the hour is to find a cure for COVID-19. For this, we need to figure out how the virus behaves under various circumstances. Artificial intelligence can help conduct millions of tests on the virus in a relatively small amount of time. We also need to test various drugs, and see how well they perform. AI can be used for this, and it will save a lot of time, effort and money.

**Predicting Survival Chances**
Healthcare professionals all over the world are dealing with immense pressure, and sometimes an overwhelmingly large number of patients to take care of. Machine Learning classification algorithms, and AI technology can help predict survival and recovery chances of patients, which can help doctors treat their patients more efficiently.

**Logistics and transport**
As a preventive measure against the spread of the disease, many nations are under lockdown. This threatens the supply chain, i.e., the goods chain from farms and factories to the markets. To ensure the availability of essential goods, and especially perishable goods, to consumers in time, various optimization techniques are being used.

Thus, technology is playing its part in fighting the spread of COVID-19. It is aiding healthcare professionals and authorities in ways that we couldn't even have imagined the existence of, a century ago.

# Approximation of the Value of $\pi$

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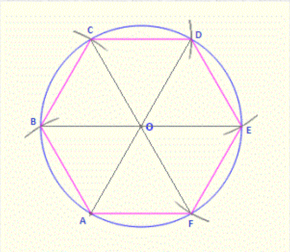
One of the many known transcendental numbers is the irrational constant *pi* represented by the symbol $\pi$.
Now, before beginning, it must be mentioned that a circle may be approximated to a polygon with $n$ number of sides of equal lengths such that $n$ approaches infinity.

**Method of calculation used:**
(throughout the text, sqrt(a)= square root of number a)

The method used to calculate the value of $\pi$ involves considering a unit circle. We know that the formula for calculating the area of a circle is $Area = \pi(radius)^2$. Here, if we consider a unit circle (circle of 1 unit radius) then we have $area = \pi\ unit^2$. Put simply, without the dimensions of area, the equation becomes $area = \pi$.

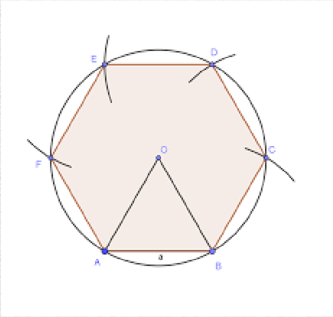
Now, we may construct a unit circle and inscribe a regular hexagon inside this circle of unit radius such that the vertices of the hexagon are points on the circle (this may be done by using a compass and constructing the hexagon thereon by cutting arcs on the circle).

For example, in the figure below ABCDEFA is a regular hexagon inscribed in a circle.



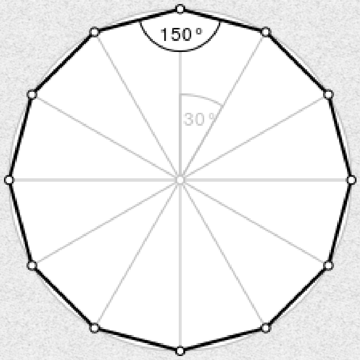

Then, one can calculate the value of the area of this hexagon using the formula $3\sqrt{3}\times$*(length of a side)*$^2/2$, which comes out to be $3\sqrt{3}/2$ (since the length of side is one unit, as it coincides with the radius of the circle), which is approximately 2.6 in value. Next, we inscribe a 12-sided polygon in the circle such that its vertices lie on the circle.

Construction: For simplicity, we may consider only one side of the hexagon and connect its vertices on the circle to the center of the circle. We have considered an equilateral triangle since we are concerned with only the value of the area and not the whole figure. Moreover, a hexagon can be considered to be made out of six equilateral triangles.




Thus, we consider the triangle OABO, as shown in the figure above.
Next, we join the perpendicular bisector of that side of the considered equilateral triangle which acts as a chord of the circle (chord AB in the above triangle). The point of intersection of this bisector on the circle is a vertex of the 12-sided polygon we aim to construct. This constructed perpendicular bisector connects this vertex to the center of the circle, by the *perpendicular bisector of a chord* conjecture. We see that we have obtained an isosceles triangle on either side of this bisector (each having two sides coinciding with radii of the circle). As done before, we may consider either one of the two adjacent isosceles triangles.

This 12-sided polygon is formed by placing 12 of each of these triangles next to each other such that any two adjacent triangles share one side in common, which is not the base.




Here, we will consider one of these acute isosceles triangles.
Next, the length of a side of this regular 12-sided polygon (we consider only one section of the polygon, that is, one isosceles triangle) may be calculated by using the Pythagoras Theorem. This length is the base of the triangle (which is also a chord of the circle). Similarly, height of the isosceles triangle is calculated by using the same theorem, by taking into consideration the height of the initially considered equilateral triangle of the hexagon.
After calculating the values of height and base of the isosceles triangle, one may calculate the area of the triangle by the formula

$$\textit{Area of Triangle} = \textit{(base length)}\times\textit{(height length)/2}$$

Since the complete figure is that of a 12-sided polygon in actuality, the area of this polygon is *12×(area of one triangle)*. Calculations show that the value of this area is equal to 3.

Following the same process, one can go on constructing 24-sided, 48-sided, 96-sided, 192-sided, 384-sided, etc. polygons and calculate their areas.

Depending on the person, one is at a liberty to choose any $n$ sided polygon, construct its inscription inside the unit circle and make the $2n$ sided polygon, $4n$ sided polygon, etc. by the above mentioned method or any other method. Here, we have constructed a hexagon, then a 12-sided polygon, then a 24-sided polygon and so on, since their construction is easy with the help of a compass, a scale and a pencil, and their areas can be easily calculated thereon.

As one increases the number of sides of the polygons and calculates their areas, he/she draws himself/herself closer to the value of $\pi$, since as the number of sides increases, the figure gradually begins to assume the shape of a circle, as mentioned before. Moreover, the radius here is one, implying that $area = \pi$.

Listing down the values of areas obtained:

| **Polygon – Area** | **Approximate value of the calculated area** |
|---|---|
| Hexagon - $3\sqrt{3}/2$ | 2.6 |
| 12-sided polygon - 3 | 3 |
| 24-sided polygon - $6\sqrt{2-\sqrt{3}}$ | 3.106 |
| 48-sided polygon – $12\sqrt{(2-\sqrt{2+\sqrt{3}}}$ | 3.1326 |
| 96-sided polygon – $24\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}$ | 3.1394 |
| 192-sided polygon – $48\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}}$ | 3.1140 |
| 384-sided polygon – $96\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}}}$ | 3.14145 |

One may be able to observe a pattern developing here, which would lead to the value of $\pi$ eventually.

# Fractals

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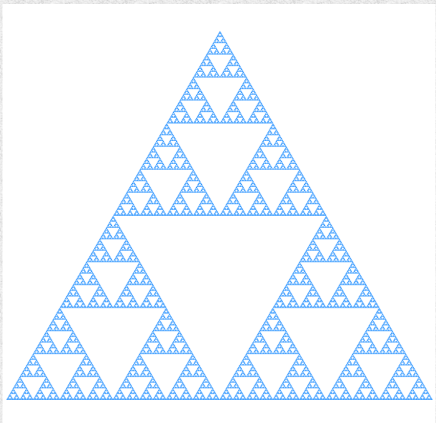
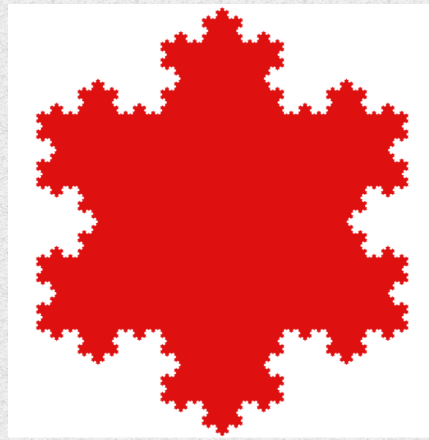
*Fractals* are some of the most beautiful and bizarre geometrical shapes. These shapes look the same at different scales. A small part of the shape looks the same as the whole, and this property is called *self-symmetry.*

One can construct a fractal by drawing a simple pattern again and again for reducing scales. But this process becomes complicated when the scale becomes too small to draw. So using mathematics, properties of real fractals can be predicted. Two famous fractals are:



(a) Sierpinski Gasket
The Sierpinski Gasket is created by repeated removal of the triangles that are constructed by joining mid-points of each side of a triangle.



(b) von Koch Snowflake
The von Koch Snowflake can be created from a triangle by recursively adding a similar triangle to every edge of the triangle.

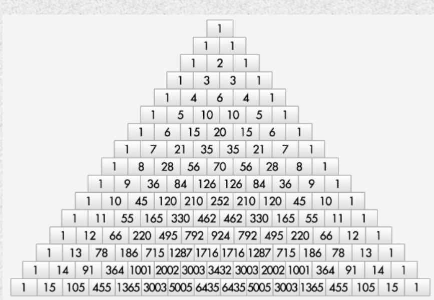
The term "*fractal*" originates from the fact that these (mathematical) objects do not have a whole dimension: instead, they have a *fractional* dimension. When a shape of dimension $d$ is changed by a factor $x$ then the properties of the shape (length, area, volume) change by the factor of $x^d$.
But when the Sierpinski Gasket is scaled by a factor of two, its area triples. So the dimension of the Sierpinski Gasket is in fact $d = log_2 3$.
When the von Koch Snowflake is scaled by a factor of three, its length quadruples. So the dimension of the von Kosh Snowflake is $d = log_3 4$.
The dimensions of these shapes are fractional. Thus they are called *fractals.*

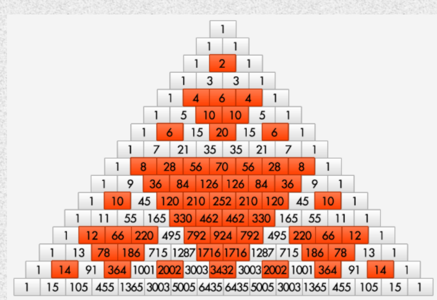
## The Sierpinski Gasket

The Sierpinski Gasket consists of an infinite number of triangles which have been removed from the centres of the other triangles. There are other ways to create this shape:

1. Pascal's Triangle

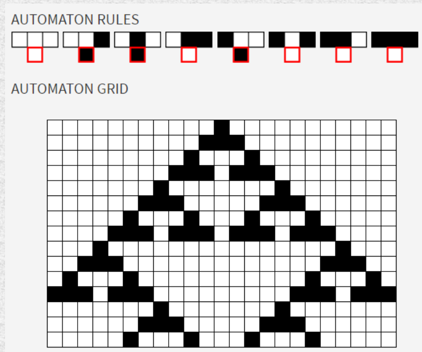
   A Sierpinski Gasket can be created using a Pascal's triangle by highlighting all even numbers in the triangle. This method shows that the even numbers in the Pascal's triangle are in the centre of a triangle of odd numbers.



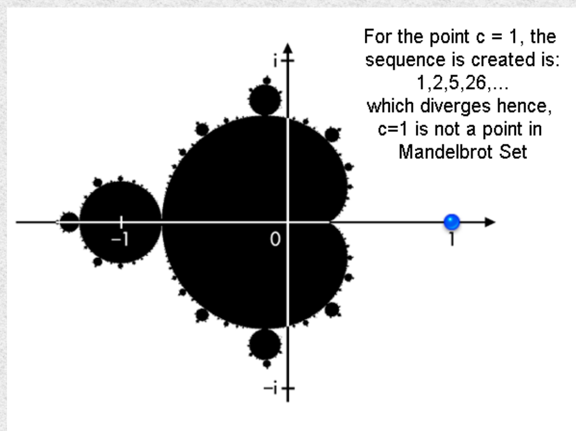
2. The Cellular Automaton

   The cellular automaton is a grid of black and white squares. Starting with one black square in the first row and applying the automaton rules as shown in the figure below, a Sierpinski Gasket can be created.




## The Mandelbrot Set

The **Mandelbrot Set** is one of the most famous fractals and is named after the French mathematician *Benoît Mandelbrot*. It resembles a person with a head, body and two arms when rotated by 90°.




The Mandelbrot Set

Consider a point $c$ on the complex plane.

- Form a sequence starting with $z_0 = 0$, where every subsequent number is given by $z_{n+1} = z_n^2 + c$. Here, $z_n$ is the $n^{th}$ number in the sequence generated for the complex number $c$.
- If the sequence diverges, then mark the point $c$ white. However, if the sequence doesn't diverge, then mark it black.

This procedure is repeated for all the points $c$ in the complex plane. The graph obtained by this is shown in the above figure. The black points in the graph make the *Mandelbrot Set.*
A computer can do these computations very quickly for millions of numbers; all pixels on a screen, for instance. The code required is simple, but the resulting fractal is unbelievably complex.

In the early 1920s, there were no computers to create this image. But using mathematics, Mandelbrot was able to predict the properties of this shape and find the pattern in this shape. The first image of the *Mandelbrot Set* was generated by an IBM supercomputer in 1980.

## Fractals in Nature and Real Life

*Fractals* cannot exist in nature because reducing the scale/zooming in eventually ends when it comes to atoms and molecules resulting in the termination of the process. However, there are some objects in the nature which show similar properties as those of fractals. Although these shapes look completely random, there is an underlying pattern that determines how

(a) Mountain Ranges

(b) Cauliflower

(c) Snowflake

these are made. Mathematics helps us in the better study of these shapes as well as their applications in medicine, geology, meteorology, etc.

The von Koch Snowflake is created using fractals. A similar process can be used to create graphical images, textures of video games. The below image has entirely been generated by the use of fractals.

This process can also be reversed and used for image compression. Typically, images are stored by remembering the colour of each pixel, which requires a lot of memory. But by using fractals, patterns can be found for the image pixels, all the pixels with the same colour can be assigned one memory location, and the colour stored there. This reduces the space occupied by the image, and hence the image is compressed.

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[1] Mathigon.*Fractals*.URL; `www.mathigon.org/course/Fractals`

# Spanning Tree

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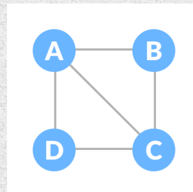
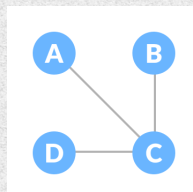
A spanning tree can be defined as a sub-graph of a connected, undirected graph $G$, that is, a tree produced by removing the desired number of edges from a graph. Now, let's understand the meaning of the words "undirected" and "connected".

An *undirected* graph is a graph in which the edges do not point in any direction. (i.e the edges are bidirectional).

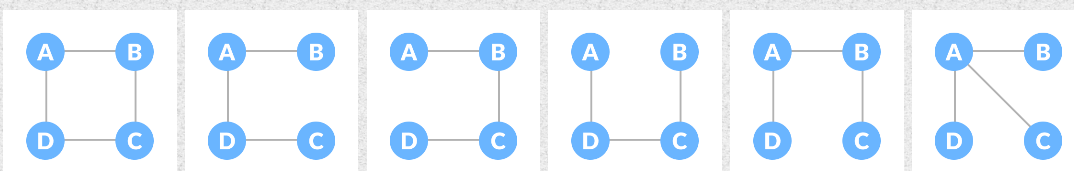



A *connected* graph is a graph in which there is always a path from a vertex to any other vertex.




If there is a vertex to which there is no path from some other vertex, it cannot be a spanning tree.
**Complete graph**: In the mathematical field of graph theory, it is a simple undirected graph in which every distinct pair of vertices is connected by a unique edge. These had previously been termed as universal graphs. To check whether a graph is complete or not, we have to check whether it has exactly $V(V-1)/2$ edges, where $V$ is the number of vertices.

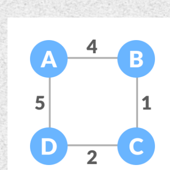
Total number of spanning trees with n vertices that can be created from a complete graph is equal to $n^{n-2}$.
Let us understand spanning trees with the help of an example.



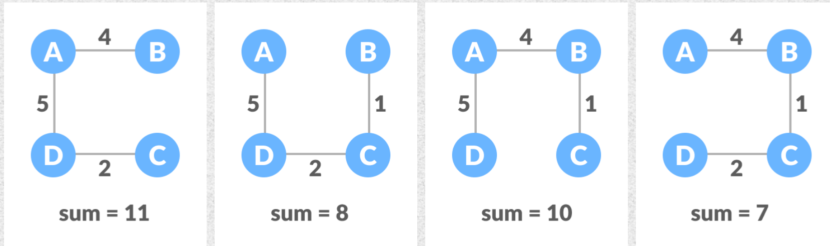

We have $n = 4$. Thus, the maximum number of possible spanning trees is equal to 16. Thus, 16 spanning trees can be formed from the above graph.
Now, let us talk about minimum spanning trees.

A minimum spanning tree is a spanning tree in which the sum of the *weights* of the edges is the least among all the possible spanning trees.

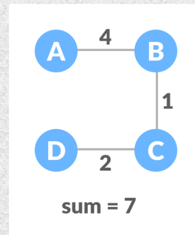
**What does the term 'weighted graph' mean?**
A weighted graph is a graph in which each branch or edge is given a numerical value (referred to as 'weights'). Mostly, algorithms prefer non-negative, positive and integral values.
Let us go through this with the help of an example.
The given graph is:



The possible spanning trees for the above graph are:




The minimum spanning tree from amongst the above spanning trees is:




**Now, the question arises, what is the need of a spanning tree?**
They are primarily used for computer network routing protocols, cluster analysis and civil network planning.

**What can we say about minimum spanning trees?**
They are primarily used to find the best paths using maps, and to design networks like telecommunication networks, water supply networks and electrical grids.
In particular, minimum spanning trees have a direct application in the design of networks. They are used in algorithms approximating the travelling salesman problem, the multi-terminal minimum cut problem and minimum cost (weight) perfect matching. Other practical applications include cluster analysis, handwriting recognition and image segmentation.

There are two famous Algorithms for finding minimum spanning trees:

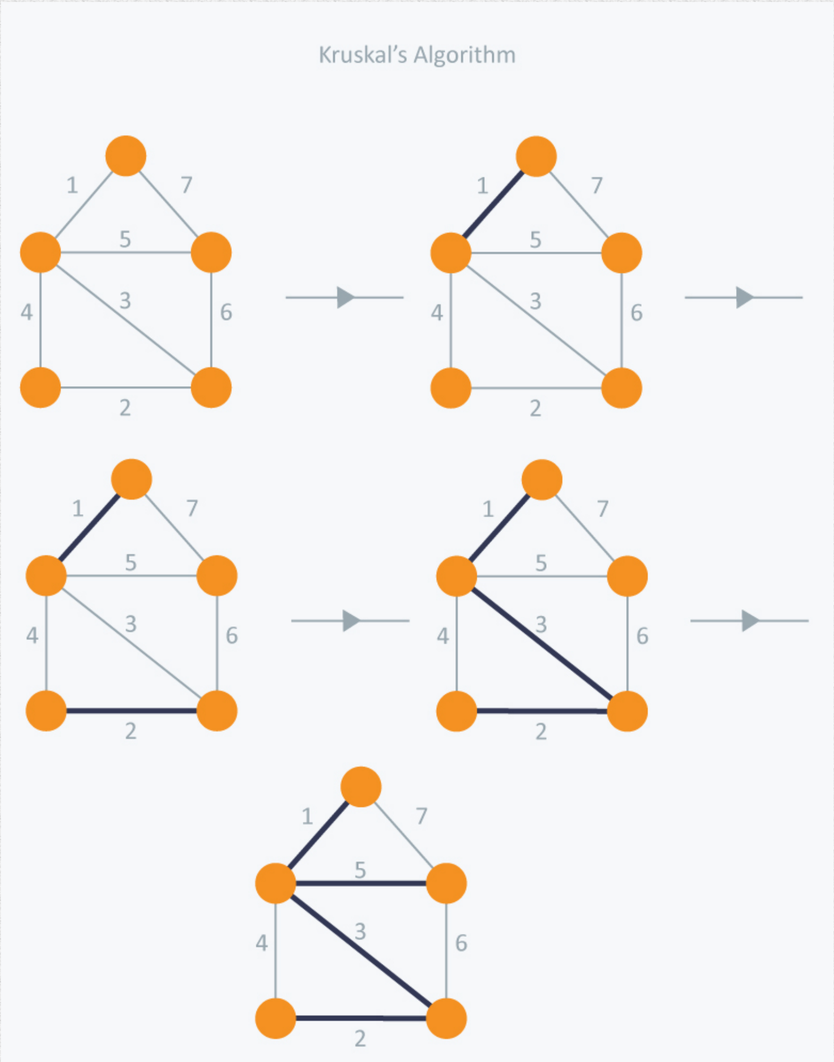
1. **Kruskal's Algorithm**
2. **Prim's Algorithm**

First, we will talk about *Kruskal's algorithm.*
Kruskal's algorithm builds the spanning tree by adding edges one by one onto a growing spanning tree. Kruskal's algorithm follows the greedy approach as in each iteration, it determines the edge which has the least weight and adds it to the growing spanning tree.
**Algorithm Steps:**

- Sort the graphs edges with respect to their weights.
- Start adding edges to the Minimum Spanning Tree from the edges with the smallest weight until the edge with the largest weight.
- Only add edges which do not form a cycle but connect only disconnected components.

**How do we check if two vertices are connected or not?**
We can easily figure it using concept of "*disjoint sets.*" As we all know, disjoint sets are the sets whose intersection is empty. This means that they don't have any elements in common.

For this, consider an example:
In Kruskal's algorithm, in each iteration, we will select the edge with the lowest weight. So, we will start with the lowest weighted edge first, i.e., the edge with weight 1. After that, we will select the second lowest weighted edge, i.e., the edge with weight 2. Notice that these two edges are totally disjoint. Now, the next edge will be the third lowest weighted edge, i.e., the edge with weight 3, which connects the two disjoint pieces of the graph. Now, we are not allowed to pick the edge with weight 4 because that will create a cycle and we cannot have any cycles. So, we will select the fifth lowest weighted edge, i.e., edge with weight 5. We will ignore the other two edges because they will create cycles. In the end, we end up with a minimum spanning tree with total cost $11 (= 1 + 2 + 3 + 5)$.




Now, we will talk about the other algorithm, i.e., *Prim's Algorithm*.

This also follows the greedy approach to find the minimum spanning tree. In this algorithm, we grow the spanning tree from the starting position.

**What is the difference between the two?**

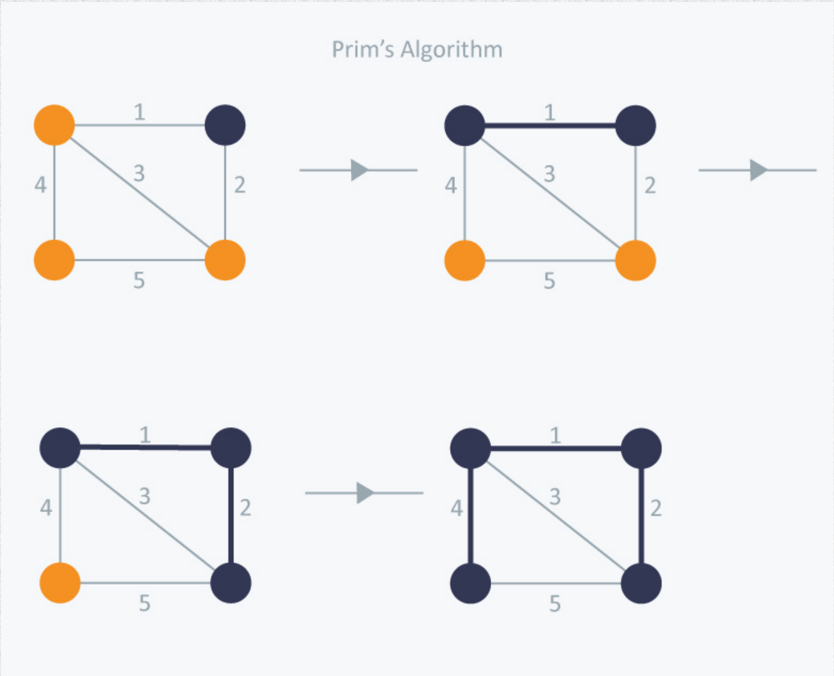
Unlike adding an edge in Kruskal's, *we add vertices to the growing spanning tree* in Prim's Algorithm.

**Algorithm steps:**

- Maintain two disjoint sets of vertices; one containing vertices that are in the growing spanning tree and the other with vertices that are not in the growing spanning tree.
- Select the cheapest vertex that is connected to the growing spanning tree and is *not* in the growing spanning tree. Add it into the growing spanning tree. This can be done using priority queues. Insert the vertices that are connected to the growing spanning tree into a priority queue.
- Check for cycles. To do that, mark the nodes which have been already selected and insert only those nodes into the priority queue that are not marked.

For better understanding, consider the example below.

In Prim's Algorithm, we will start with an arbitrary node (it does not matter which one) and mark it. In each iteration, we mark a new vertex that is adjacent to the one that we have already marked. Since it is a greedy algorithm, Prim's algorithm will select the cheapest edge and mark the vertex. So, we will simply choose the edge with weight 1. In the next iteration, we have three options: edges with weights 2, 3 and 4. So, we will select the edge with weight 2 and mark the vertex. Now again we have three options: edges with weight 3, 4 and 5. But we cannot choose the edge with weight 3 as it will create a cycle. So, we select the edge with weight 4 and end up with the minimum spanning tree of total cost $7(= 1+2+4)$.




## Definitions of Words Used in the Text:

**Graph**: It is a structure amounting to a set of objects in which some pairs of the objects are in some sense "related". A graph is a pair $G = (V, E)$, where $V$ is a set whose elements are called *vertices* and $E$ is the set of two-sets of vertices, whose elements are called *edges*.

**Sub-Graph**: It is a graph whose vertex set is a subset of the vertex set and edge set is a subset of the edge set of a graph set. Then, the former graph is a sub-graph of the latter.

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# The Fields Medal

*Prakruti Kalsaria,*
*Applied Mathematics and Humanities Department, SVNIT*

The Fields Medal is one of the most prestigious and recognized prizes in the field of mathematics. It is awarded, once every four years, to two to four mathematicians under 40 years of age at the quadrennial International Congress of the International Mathematical Union (IMU). The prize is awarded for exceptional contributions towards mathematics. According to the annual Academic Excellence Survey by Academic Ranking of World Universities (ARWU), this medal has been consistently adjudged as the top award in the field of mathematics worldwide. In another reputed survey conducted by International Roaming Expert Group (IREG) in 2013–14, the Fields Medal finished a close second after the Abel Prize as the most prestigious international award in mathematics.

The Fields Medal was named in the honour of a Canadian mathematician, John Charles Fields. The physical medal is made up of 14 ct gold, has a diameter of 63.5 mm and weighs about 169 g. Its unit price is approximately CA$5500. Canadian sculptor R. Tait McKenzie had designed the medal.

- On the obverse:
  1. The head represents Archimedes facing right.
  2. A quote attributed to him, in Latin: *"Transire suum pectus mundoque potiri"* which means "Rise above oneself and grasp the world".
  3. In Greek capitals, the word $APXIMH\Delta O\Upsilon\Sigma$ meaning 'of Archimedes'.
  4. The artist's monogram and date RTM, MCNXXXIII (which contains an error as it should be 'MCMXXXIII, for 1993).

Obverse of Medal

- On the reverse is the inscription (in Latin):
  *"CONGREGATI*
  *EX TOTO ORBE*
  *MATHEMATICI*
  *OB SCRIPTA INSIGNIA*
  *TRIBUERE"*
  It means: "Mathematicians congregated from the entire world have awarded (this medal) because of outstanding writings."

Reverse of Medal

The medal comes with a monetary award of CA$15000. The medal was first awarded in 1936 and has been awarded every four years since 1950. The Iranian mathematician Maryam Mirzakhani became the first female Fields Medallist in 2014. A trust established by JC Fields at the University of Toronto funds the medals and cash prizes and has been supplemented periodically, but is still significantly underfunded.

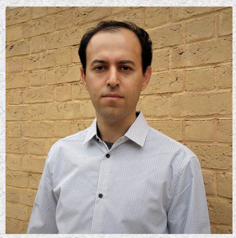
The Fields Medal Committee is chosen by the Executive Committee of the International Mathematical Union and is typically chaired by the IMU President. Until the award ceremony, the names of members of the committee remain anonymous except that of the chair of the committee.

On August 1, 2018, at the opening occasion of the IMU International Congress, the most recent group of Fields Medallists received their awards. The ceremony was held in Rio de Janeiro, Brazil.

The laureates who were honoured were:

- **Caucher Birkar**

  He is a UK-based Iranian Kurdish mathematician known for his works in algebraic geometry. His major contribution has been in the field of modern birational geometry. He is a professor at the University of Cambridge, UK. According to the short citation provided by the committee, Caucher Birkar was awarded the Fields Medal "for the proof of the boundedness of Fano varieties and for contributions to the minimal model program."



Caucher Birkar

- **Alessio Figalli**

  He is an Italian mathematician whose work is primarily on calculus of variations and partial differential equations. He is a chaired professor at ETH Zürich. He was awarded the Fields Medal 2018 "for his contributions to the theory of optimal transport, and its applications to partial differential equations, metric geometry, and probability."

Alessio Figalli

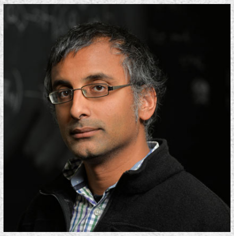
- **Peter Scholze**

  He is a German mathematician and has been a professor at the University of Bonn since 2012, and Director of the Max Planck Institute for Mathematics since 2018. He is known for his work in algebraic geometry and is regarded as one of the leading mathematicians in the world. He won the Fields Medal 2018 "for transforming arithmetic algebraic geometry over $p$-adic fields through his introduction of perfectoid spaces, with application to Galois representations, and for the development of new cohomology theories."

Peter Scholze

- **Akshay Venkatesh**

  He is an Australian mathematician of Indian origin and a professor at the School of Mathematics at the Institute for Advanced Study, Princeton, since August 2018. The fields of counting, equidistribution problems in automorphic forms and number theory, in particular representation theory, locally symmetric spaces, ergodic theory, and algebraic topology are included in his research interests. The Fields Medal 2018 was awarded to him "for his synthesis of analytic number theory, homogeneous dynamics, topology, and representation theory, which has resolved long-standing problems in areas such as the equidistribution of arithmetic objects."



Akshay Venkatesh

# A Brief History of Transcendental Numbers

*Nitish Kumar Dubey,*
*Applied Mathematics and Humanities Department, SVNIT*

The numbers are how most of us introduced ourselves with mathematics. Then came the operations of addition, subtraction, multiplication, and division that together comprise *Algebra.* Algebra is often regarded as a mindless machine which produces some output for a given input and we have to make our physical sense out of it. While it's true, in our daily lives, we have associated the numbers and our arithmetic such that for an instant if I say that "there is a number which we can't count to using our arithmetic" (I am not talking about infinity or numbers those are too big to even imagine) is a statement that doesn't makes sense; it becomes more counter-intuitive when we postulate that the number of these kind of numbers is more than the numbers which we can count to.

## Greeks and Transcendental numbers:

One of the greatest mathematical problems is the problem of *squaring the circle.* It is the challenge of *constructing a square with the same area as a given circle by using only a finite number of steps with a compass and a straightedge.* This problem was proposed by the geometers of ancient Greece. This problem was proved impossible to solve by using the transcendence of $\pi$ in 1882.

## Birth of Modern Definition:

The modern definition of Transcendental Numbers is as follows:

**Definition.** *A transcendental number is a complex number that is not an algebraic number — that is, it is not a solution of a non-zero polynomial equation with integer coefficients.*

The notion of transcendence was introduced by Leibniz when he demonstrated that the $\sin x$ was not an algebraic function, meaning it can't be defined as the root of a polynomial equation. Later on, Euler provided us with a definition that first talked about numbers,in context of transcendence.
Consider the following definition Euler had provided for a function:

**Definition.** *A function of a variable quantity is an analytic expression composed in any way whatsoever of the variable quantity and numbers or constant quantities. [Euler, 1748]*

As we can see that this definition doesn't explicitly talk about the definition of Transcendental numbers.It's been a topic of debate in mathematical community - whether Euler was aware of the existence of transcendental numbers. This definition was used very loosely by Euler,he never defined the transcendental numbers explicitly.
We cannot really find an explicitly strong definition that claims Existence of Transcendental numbers before the Lambert's conjecture of 1768 which claimed that $\pi$ was not algebraic.
In 1844 *Joseph Liouville* first proved the existence of Transcendental numbers and in 1851 he presented the first known Transcendental numbers are which are now known as "Liouville Numbers"

### Liouville Numbers:

A Liouville number is a real number $x$ with the property that, for every positive integer $n$, there exist infinitely many pairs of integers $(p, q)$ with $q > 1$ such that

$$0 < \mid x - \frac{p}{q} \mid < \frac{1}{q^n}$$

or we can write them as

$$x = \sum_{i=1}^{\infty} \frac{a_i}{b_{i!}}$$

In the special case when $b = 10$, and $a_k = 1$, for all $k$, the resulting number $x$ is called *Liouville's constant:*

$$L = 0.110001000000000000000001000000000000000000000000000000000000000000000...$$

By changing the values of a and b we can create infinite Transcendental numbers. it is impossible to find an algebraic polynomial with rational coefficients that can generate this number.
Liouville's method is indeed a way to create Transcendental numbers. However, the method gives no insights on proving the transcendence of conjectured numbers. The first number to be proven transcendental without having been specifically constructed for the purpose of proving transcendental numbers' existence was $e$, by Charles Hermite in 1873 The following year George Cantor proved that algebraic numbers are countably infinite, it was known that the Real numbers are uncountably infinite, proving that transcendental numbers are uncountably infinite. George Cantor provided a new method to map numbers showing that the size of set consisting real numbers is equal to size of set of transcendental numbers.
In 1882, Ferdinand von Lindemann published the first complete proof of the transcendence of $\pi$ In this proof, he proved that $e^a$ is not an algebraic number provided that "$a$" is an algebraic number, and then came the famous Euler's identity into play. We know that,

$$e^{i\pi} = -1$$

as we have established that $e^a$ shouldn't be algebraic if "a" is algebraic since, -1 is an algebraic number(namely, root of equation x+1 =0). Lindemann's result tells us that $i\pi$ can't be algebraic .
$i$ is root of an equation with integer coefficients

$$x^2 + 1 = 0$$

which indicates that i is an algebraic number.
Since multiplication and addition of finite algebraic numbers always result in an algebraic number(That's why we can express them as a solution of polynomials.). we can conclude that $\pi$ is not algebraic number. This also proves the impossibility of solving *squaring the circle* problem.
As our mathematical prowess has been evolved through the centuries despite knowing transcendental numbers are more than algebraic numbers. We don't have a way to locate them; we have very few numbers that are proven to be transcendental. While knowing that we are in the dark regarding these numbers we can keep David Hilbert's words in mind.

We must Know,
We will Know.

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# Bowling and its Mathematics

*Rutvij Tole and Parth Parmar,*
*Applied Mathematics and Humanities Department, SVNIT*

"The spinner continues in his spell.... over-pitched, and slog swept over deep square leg for a maximum. Next delivery.... a flighted delivery, the batsman dances down the pitch and misses.... He's out! Stumped by the wicket-keeper!"

Spinners are an important asset for a captain and for the team. They may get belted away for sixes but they are also the x-factors for the team. Now, you might be wondering why I am writing cricket commentary in a mathematics article. Well, it is related to mathematics.

In a game of cricket, there are two main points of interest as far as the flight of the ball is concerned. The first is the time from the release of the ball by the bowler, to when it is either hit or missed by the batsman. The second is the time after the collision of the ball with the bat. As the batsman's goal is to score as many runs as possible, most shots are played so that the ball remains close to the ground, and is therefore harder to catch by a fielder.

The bowler's main aim is to pitch the ball so the batsman does not hit the ball to the best of his ability. The flight path of the ball is such that the trajectory can be represented by a simple equation. However, this does not necessarily apply to slow pitches. There is a small set of critical speeds in which pressure imbalances cause the ball to swing (deviate) to one side or the other. These speeds are functions of several variables, including the angle of the seam, surface texture of the ball, the spin put on the ball by the bowler, and the air currents. Forces up to 30% of weight of the ball push the ball from the side. In the horizontal direction of motion,

$$m\left(\frac{dv}{dt}\right) = -kv^2$$

where $m$ is the mass of the ball, $\left(\frac{dv}{dt}\right)$ is the derivative based on time, representing acceleration, and $k$ is the side force constant. This equation is only true if the vertical motions are completely ignored. If this equation is changed to be a derivative of velocity with respect to distance rather than time, it will be:

$$v\left(\frac{dv}{dx}\right) = -\frac{k}{m}v^2$$

where all variables remain the same, but $x$ is the distance of the ball from the ground. This equation can be solved to give

$$x = \left(\frac{m}{k}\right)\ln\left(\frac{v_0}{v}\right)$$

where $ln$ is the natural logarithm, $v_0$ is the initial velocity, and all other variables remain constant. This shows the relationship of distance and velocity after a hit by the bowler. In order to find an estimate of the time of flight, separation of variables can be performed on the last equation to give

$$t = \left(\frac{m}{k}\right)\left\{\left(\frac{1}{v}\right) - \left(\frac{1}{v_0}\right)\right\}$$

This shows how long the ball is in the air for a particular velocity. Once each of these equations is solved using the known variable(s), the deviation of the ball from the visible path can be traced. Even the slightest variation can trick a batter's eye into missing the ball or mistiming a shot.

### Trajectory of a Ball

The trajectory can be calculated by considering all the possible forces acting on the ball after being released by the bowler. This is how it goes. Baker [**1**] set out equations for the flight of both compact debris (with the three length dimensions broadly comparable with each other) and sheet debris (with two major and one minor length dimension) under extreme wind conditions, such debris having recently been recognized as one of the major causes of building damage in severe wind storms. These winds accelerate the debris from zero velocity to velocities close to the wind velocity. In Quinn et al. [**2**], the authors utilize the compact debris equations further to investigate the flight of ballast under high-speed trains, an issue that is becoming to be of increasing concern for high-speed train operators. This involved the specification of the air velocity underneath the trains. Again, the air movements under the train accelerate the ballast from stationary to velocities close to the train-induced air velocities. In studying cricket ball trajectories, it is appropriate to again use the compact debris equations, but with the difference that this time the object under consideration is moving much faster than the prevailing

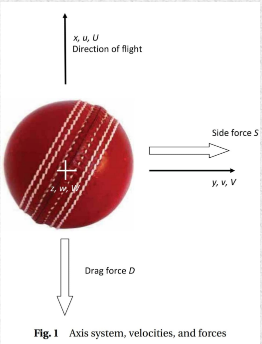

Fig. 1 Axis system, velocities, and forces

Fig. 1

air velocities due to natural wind. Applying Newton's Law in the $x$ (along the pitch), $y$ (across the pitch), and $z$ (vertical) directions and suitably non-dimensionalizing, one obtains the basic trajectory equations for cricket balls.

$$\frac{d\bar{u}}{dt} = -\left[(\bar{u}-\bar{U})^2 + (\bar{v}-\bar{V})^2 + (\bar{w}-\bar{W})^2\right]^{0.5} \times (\bar{u}-\bar{U})CT \tag{1}$$

$$\frac{d\bar{v}}{dt} = -\left[(\bar{u}-\bar{U})^2 + (\bar{v}-\bar{V})^2 + (\bar{w}-\bar{W})^2\right]^{0.5} \times \left[-C_D T(\bar{v}-\bar{V}) + C_S T(\bar{u}-\bar{U})\right] \tag{2}$$

$$\frac{d\bar{w}}{dt} = -\left[(\bar{u}-\bar{U})^2 + (\bar{v}-\bar{V})^2 + (\bar{w}-\bar{W})^2\right]^{0.5} \times (\bar{w}-\bar{W})C_D T - 1 \tag{3}$$

Here $\bar{u}$, $\bar{v}$, and $\bar{w}$ are the cricket ball velocities in the $x$, $y$, and $z$ directions, respectively, non-dimensionalized by dividing by the initial speed at which the ball is bowled, q. $\bar{t}$ is the non-dimensional time from the release of the ball tg/q. $\bar{U}$ , $\bar{V}$ , and $\bar{W}$ are the wind velocities in the x, y, and z directions, respectively, again normalized with q. $C_D$ and $C_S$ are the aerodynamic drag and side coefficients defined as

$$C_D = \frac{D}{0.5\rho A Q^2} \tag{4}$$

$$C_S = \frac{S}{0.5\rho A Q^2} \tag{5}$$

Where $D$ and $S$ are the drag and side forces, respectively, $A$ is the ball cross-sectional area, $\rho$ is the density of air, and $Q$ is the total relative velocity of the ball relative to the air $\left[(\bar{u}-\bar{U})^2 + (\bar{v}-\bar{V})^2 + (\bar{w}-\bar{W})^2\right]^{0.5}$. $T$ is the Tachikawa number **[3]** given by,

$$T = \frac{\rho A q^2}{2mg} \tag{6}$$

where $m$ is the mass of the ball.

This represents the ratio of the initial flow inertia force on the ball to the weight of the ball. It can thus be seen from equations (1) to (3) that the governing parameters of the problem are the product side and drag force coefficients, and the Tachikawa number. The axis system and the aerodynamic forces are illustrated in Fig. 1.

**No wind case**

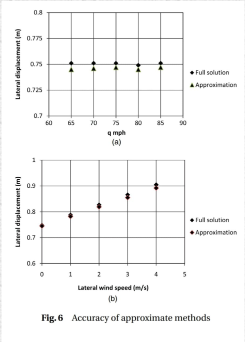

Fig. 6 Accuracy of approximate methods

In this section an approximate solution to the trajectory equations for the no-wind condition (i.e.$\vec{U} = \vec{V} = \vec{W} = 0$), and also for the case where the drag and the side force coefficients are constant (i.e., on the sub- and supercritical branches of the generic force coefficient distributions) of Fig. 5 is considered. We assume that the lateral and vertical ball velocities are much smaller than the longitudinal velocities. In this case, the trajectory equations reduce to

$$\frac{d\vec{u}}{dt} = -C_D T \vec{u}^2 \tag{7}$$

$$\frac{d\vec{v}}{dt} = -C_S T \vec{u}^2 \tag{8}$$

$$\frac{d\vec{w}}{dt} = -1 \tag{9}$$

These are easily integrable, analytically, and result in the following expressions for the dimensionless ball velocities and displacements in terms of dimensionless time.

$$\vec{u} = \frac{1}{1 + C_D T \bar{t}} \tag{10}$$

$$\vec{v} = \left(1 - \frac{1}{1 + C_D T \bar{t}}\right) \frac{C_S}{C_D} \tag{11}$$

$$\vec{w} = \sin\alpha - \bar{t} \tag{12}$$

$$\bar{x} = \frac{1}{C_D T} \log(1 + C_D \bar{t}) \tag{13}$$

$$\bar{y} = \left(t - \frac{1}{C_D T} \log(1 + C_D T \bar{t})\right) \frac{C_S}{C_D} \tag{14}$$

$$\bar{z} = \bar{t} \sin\alpha - \frac{\bar{t}^2}{2} \tag{15}$$

Here $\alpha$ is the initial angle of inclination in the $x$–$z$ plane (i.e., the angle to the vertical at which the ball is bowled). Note that the lateral and vertical trajectories are effectively independent of each other, with only the latter being a function of the initial angle $\alpha$. Now, eliminating the dimensionless time between the longitudinal and lateral displacement equations (13) and (14) give the following expression for the lateral trajectory.

$$\bar{y} = \frac{C_S}{{C_D}^2 T^2} (e^{C_D T \bar{x}} - 1 - C_D T \bar{x}) \tag{16}$$

Now the product $C_D T \bar{x}$ can be shown to be of the order of 0.1 to 0.2 and thus equation (16) approximates to

$$\bar{y} = \frac{C_S T}{2} \bar{x}^2 \tag{17}$$

Figure 6(a) shows a comparison between the displacement predicted using equation (17) and those predicted from a full solution of equations (1) to (3) for the new ball, sub-critical $Re$ case for a range of bowling speeds. The ball mass is taken as 0.16 kg, and the ball diameter as 0.072 m. The lateral displacements are given at a distance of 18 m from the bowling position – roughly corresponding to the position of the batsman. Agreement can be seen to be good, with differences between the approaches of the order of a few millimetres (note the large y-axis scales), and a close examination of the results confirms the approximations made above concerning the small values of the lateral and vertical velocities and the product $C_D T \bar{x}$. It can thus be seen that, as a good approximation, the dimensionless trajectory is parabolic, with the constant of proportionality being the product of the side force coefficient and the Tachikawa number. From the definitions of these parameters given in equations (4) to (6), this can be seen to be proportional to the ratio of the side force to the weight of the cricket ball – the so-called 'swing force ratio' of reference **[4]**, also used in the presentation of results in the various works of Mehta. Perhaps more remarkably, if one substitutes the definitions of Tachikawa number from equation (6) and expresses the distances in dimensional terms, equation (17) becomes

$$\bar{y} = \frac{C_S \rho A}{4m} \bar{x}^2 \tag{18}$$

**Cross Wind Case**

Now, consider a similar approach for the case of a small crosswind (i.e. $\bar{U} = \bar{W} = 0$, and $\bar{V} << \bar{u}$). In this case the longitudinal and vertical equations of motion remain the same as equations (7) and (9), but the lateral displacement equation becomes

$$\frac{d\bar{v}}{dt} = C_D T \bar{u} \bar{v} + C_S T \bar{u}^2 \tag{19}$$

The solution to equation (19) is again quite straightforward, and results in the following approximation for the dimensionless trajectory

$$\bar{y} = \frac{(C_S + C_D\bar{V})}{2}\bar{x}^2 \tag{20}$$

This is of the same form as equation (18) and reveals that the effect of a crosswind is to increase the constant of proportionality in the parabolic trajectory equation. Figure 6(b) shows a comparison of the results of equation (20) with predicted trajectory values from equations (1) to (3) for a bowling speed of 75 mile/h and a range of wind speeds. Again, the agreement can be seen to be excellent. Note that the average hourly wind speed 10 m above the ground level in England is of the order of 4 m/s [**5**]. The variation of this velocity with height is approximately logarithmic and for a ground roughness typical of cricket pitches (around 0.01 m), it can be shown that this velocity corresponds to a velocity of 2.7 m/s at 1.0 m above the ground: the approximate average height of a cricket ball trajectory. At this speed, the average deflection due to wind effects alone is of the order of 10 cm.

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# ACHIEVEMENTS

Following is the list of achievements of students of Applied Mathematics and Humanities Department in the year 2019-2020:

1. **Vishal Agarwal (M.Sc. Mathematics 3$^{rd}$ year)**
   - Selected, along with five others, to represent Gujarat, Dadra and Nagar Haveli NCC Directorate at IMA Attachment Camp, 2019 held at Indian Military Academy, Dehradun from 12$^{th}$ to 23$^{rd}$ June, 2019.
   - Represented SVNIT in the All India Inter NIT Weight Lifting, Powerlifting and Best Physique Competition 2019-20 held at Malaviya National Institute of Technology, Jaipur in Powerlifting (74 kg category).
2. **Ashwin Verma (M.Sc. Mathematics 3$^{rd}$ year)**
   - Did Research Internship at Maulana Azad National Institute of Technology, Bhopal in the field of Machine Learning.
3. **Shashank Gupta (M.Sc. Mathematics 3$^{rd}$ year)**
   - Won Gold medal in the All India Inter NIT Weight Lifting, Powerlifting and Best Physique Competition 2019-20 held at Malaviya National Institute of Technology, Jaipur in Best Physique (under 80 kg category).
   - Won Gold medal in Inter NIT Badminton Competition 2020 held at Sardar Vallabhbhai National Institute of Technology, Surat.
4. **Ronak Sharma (M.Sc. Mathematics 3$^{rd}$ year)**
   - Represented SVNIT in the All India Inter NIT Cricket Tournament 2020 held at Sardar Vallabhbhai National Institute of Technology, Surat.
5. **Vaibhav Gupta (M.Sc. Mathematics 2$^{nd}$ year)**
   - Attended Mini-MTTS-2019 (Kalyan).
   - Won the 'Best Justification to Genre' award in Rangmanch 2019 organised by CHRD SVNIT, as the writer of a play in the Symbolic Poetry genre.
6. **Sravya Bollam (M.Sc. Mathematics 2$^{nd}$ year)**
   - Her team won the joint 1st prize in InQuest 2020, a 10-hour researchathon organised by SCOSH, SVNIT, in the Chemistry category.
7. **Gargi Patil (M.Sc. Mathematics 2$^{nd}$ year)**
   - Attended Mini-MTTTS-2019 (Kalyan).
   - Attended Winter School for Women in Mathematics-Dec 2019 (IISERTVM).
   - Ranked 3rd at Gujarat level in Madhava Mathematics Competition, 2020.
8. **Soham Saga (M.Sc. Mathematics 2$^{nd}$ year)**
   - Participated in the 1st National Functional Fitness Championship in India, held at Ahmedabad in June, 2019.
   - Received the title of Grand Champion in Fitness; recognized by the Asia Book of Records.
9. **Ankit Bhatia (M.Sc. Mathematics 2$^{nd}$ year)**
   - Represented SVNIT at Inter NIT Volleyball Tournament held at NIT Rourkela, in January 2020.
   - Represented SVNIT in the Volleyball Tournament held as part of CONCOURS 2019, a sports fest organized by Dhirubhai Ambani Institute of Information and Communication Technology, Gandhinagar.
10. **Priya Singh (M.Sc. Mathematics 2$^{nd}$ year)**
    - Mini Attended Mini-MTTS-2019 (VNIT, Nagpur).
11. **Rambabu (M.Sc. Mathematics 2$^{nd}$ year)**
    - Represented SVNIT in the All India Inter NIT Athletics Tournament, held at NIT Rourkela in January 2020, in Javelin Throw, Discus Throw and 4×100 metres Relay.

- Won Silver medal in the Discus Throw event in the same tournament.

12. **Prakruti Kalsaria (M.Sc. Mathematics 2$^{nd}$ year)**

- Ranked 3rd at Gujarat level in Prof. A. R. Rao Competition, 2019.
- Ranked 4th at Gujarat level in Madhava Mathematics Competition 2020.

On the occasion of *International Mathematics Day(14/03/2020)*, the department organised *Mathematics Colloquium*. The winners of the events conducted on that day:

- ***Poster Presentation***
  Winner: **Hitesh Bansu (PhD Scholar)**
  Runner-up: **Rishikesh Yadav (PhD Scholar)**

- ***Paper Presentation***
  Winner: **Ramkumar Radhakrishnan (M.Sc. Physics 3$^{rd}$ year)**
  Runner-up: **Aniruddha Deshmukh (M.Sc. Mathematics 5$^{th}$ year)**

- ***Cipher***
  Winners:

  **Mridul Sehgal (M.Sc. Mathematics 1$^{st}$ year)**

  **Vibhav Garg (M.Sc. Mathematics 1$^{st}$ year)**

- ***Math Quiz***
  Winners:

  **Niraj Velankar (M.Sc. Mathematics 2$^{nd}$ year)**

  **Prakruti Kalsaria (M.Sc. Mathematics 2$^{nd}$ year)**

  **Chirag Gupta (B.Tech Mechanical 1$^{st}$ year)**

- ***Model Making Competition***
  Winners:

  **Ramkumar Radhakrishnan (M.Sc. Physics 3$^{rd}$ year)**

  **Tiyasa Kar (M.Sc. Physics 3$^{rd}$ year)**

Two of our students have scored a complete 10 CGPA in the academic year 2019-20:

1. **Aniruddha Deskhmukh** (9$^{th}$ Semester)
2. **Purva Sehgal** (5$^{th}$ Semester)

The following *M.Sc. Mathematics 5$^{th}$ year* students are currently completing their dissertation outside campus:

1. **Gowri R Chandran**: *Bar-Ilan University, Israel*
2. **Sarthak Gupta:** *University of Göttingen, Germany*
3. **Manpreet Budhraja:** *IIIT Hyderabad, India*

Two of our students **Aniruddha Deshmukh** *(M.Sc. Mathematics 5$^{th}$ year)* and **Shubham Yadav** *(M.Sc. Mathematics 5$^{th}$ year)* along with **Dr. Dhananjay Gopal** *(Assistant Professor, Sardar Vallabhbhai National Institute of Technology, Surat)* and **Dr. Abhay S Ranadive** *(Professor, Ghasidas Vishwavidyalaya(A Central University), Bilaspur, Chattisgarh)* have completed a book called "An Introduction to Metric Spaces".

The book serves as an introductory material for metric spaces, which forms an essential base for studying Topology, Functional Analysis, Differential Geometry, etc. The authors of this book have kept in mind the issues that an undergraduate student might come across while dealing with the concept, and have therefore, tried their best to include as much geometrical interpretation as possible. This book can be used both as a self-study material by students or as a regular text-book for a semester long course on Metric Spaces. The book is expected to be out in the market by July, 2020.

The following students have cleared the GATE examination:

1. **Shubham Yadav (M.Sc. Mathematics $5^{th}$ year)**, *AIR-226*
2. **Aniruddha Deshmukh (M.Sc. Mathematics $5^{th}$ year)**, *AIR-279*
3. **Kaushal Naliyadra (M.Sc. Mathematics $5^{th}$ year)**, *AIR-336*
4. **Bhargavi Bhatt (M.Sc. Mathematics $5^{th}$ year)**, *AIR-416*
5. **Rahul Kamble (M.Sc. Mathematics $5^{th}$ year)**, *AIR-1840*

The following students have qualified the CSIR-NET examination held in June 2019:

1. **Aniruddha Deshmukh (M.Sc. Mathematics $5^{th}$ year)**, *AIR-25(JRF)*
2. **Rakesh Meena (M.Sc. Mathematics $5^{th}$ year)**, *AIR-125(LS)*
3. **Ekadashi Das (PhD Scholar)**, *(JRF)*

The following students have qualified the CSIR-NET examination held in December 2019:

1. **Jayesh Savaliya (PhD Scholar)**, *AIR-43(LS)*
2. **Harshad Sakariya (PhD Scholar)**, *AIR-118(JRF)*
3. **Lavuri Devilal (M.Sc. Mathematics $4^{th}$ year)**, *AIR-249(JRF)*
4. **Rakesh Meena (M.Sc. Mathematics $5^{th}$ year)**, *AIR-252(JRF)*

The following *M.Sc. Mathematics $5^{th}$ year* students have received placements:

1. **Aman Bakshi** - *Affine Analytics*
2. **Manpreet Budhraja** - *Oracle Financial Service*
3. **Anuveshika Prasad** - *Oracle Financial Service*
4. **Garvit Bhatt** - *BYJU's*
5. **Manish Kumar** - *BYJU's*
6. **Aman Bakshi** - *Asaan Jobs(Second Placement)*
7. **Pavan Kumar Saini** - *Infosys*
8. **Mahipal Upadhyay** - *Infosys*
9. **Gaurang Savani** - *Infosys*
10. **Harsh Jariwala** - *Akash Institute*

***Congratulations to All!***

# Editorial Team

**Aniruddha Deshmukh**

**Vishal Agarwal**

**Vatsal Moradiya**

**Ishika Bhatt**


**Ashwin Verma**

**Nitish Dubey**

**Prakruti Kalsaria**

**Niraj Velankar**

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